Our Heritage(UGC Care Listed)
ISSN: 0474-9030 Vol-68, Special Issue-12

National Conference on Recent Trends in Physics, Chemistry and Mathematics (RTPCM-2020)
Held on 4th February 2020 Organised by: Department of Physics, Chemistr and Mathematics, SunderraoSolanke Mahavidhyalaya, Majalgaon, MS

Number Theory

Dr. Vidya Venkatrao Deshpande

Late Laxmibai Deshmukh Mahila Mahavidyalaya, Parli-V

Email ID: vidyad469@gmail.com

ABSTRACT

Number theory studies integers, that is, whole numbers, and their relationships. Principle concepts include square numbers and higher powers of numbers, prime numbers, divisibility of numbers, greatest common divisors and the structure and number of solutions of systems of polynomial equations with integer coefficients. Many problems in number theory, while simple to state, have proofs that involve apparently unrelated areas of mathematics. A beautiful illustration is given by the use of complex analysis to prove the "Prime Number Theorem," which gives an asymptotic formula for the distribution of prime numbers. Yet other problems currently studied in number theory call upon deep methods from harmonic analysis. This article tells about Number Theory and different factors related to it.

Keyword: Number Theory, Principle concepts.

INTRODUCTION

What is Number Theory?

Number Theory, the study of the integers, is the oldest branch of pure mathematics, and also the largest. There are many questions to ask, about individual numbers and their properties, about operations on numbers, about relations between numbers, about sets of numbers, about patterns in sequences of numbers, and so on. Number Theory is famous for generating easy-to-ask, hard-to-answer questions, and that is one reason for its popularity.

Multiplication is the most interesting operation on integers. Number Theory treats factoring and divisibility, and, of paramount importance, prime numbers. The ancient Greeks knew how many primes there are, and Gauss discovered how they are distributed among the integers as a whole. Today we have a few efficient algorithms to find numbers likely to be prime, and some inefficient algorithms to factor large numbers. No one knows a formula to generate primes, nor how many "twin primes" there are.

The Encyclopedia of Integer Sequences, as of this writing, lists over 135,000 different integer sequences that are mathematically interesting. Some of the more familiar are the sequences of even numbers, odd numbers, squares, primes, triangular numbers, perfect numbers, Fibonacci numbers, and Lucas numbers. A less well known sequence is that of the Niven numbers: those divisible by the sum of their digits.

The study of number theory requires an intuition for number relationships and a facility with logic and proof.

History

The early Greek mathematicians (before 500 BCE) knew only the (positive) integers and their quotients (rational numbers). Numbers were both an area of academic study and a source of mysticism. The discovery of irrational numbers, those that could not be written as quotients of integers, was a major psychological and intellectual blow. Even after this traumatic event, the Greek mathematicians continued to place the integers in an exalted role. Plato says, "Arithmetic" [the study of numbers] "has a very great and elevating effect, compelling the mind to reason about abstract number."

Until recently, Number Theory continued to hold the same pride of place as the most beautiful, most "pure," and least applicable of the fields of mathematics. As both art and intellectual training, it has been part of education and research for thousands of years. The early 20th century mathematician G. H. Hardy took great pride in the belief that number theory was the height of both beauty and uselessness.Number Theory is a foundation of mathematics as basic as geometry and more basic than algebra. Pierre de Fermat is usually given credit for being the father of number theory.

Cryptology, the study of encoding messages that could be read only by the intended recipient, is as old as warfare and political intrigue. The obvious and historical methods relied on a two-way key. The writer of the message used the key to encode it, and the recipient used the same key to decode it. Some keys were harder than others to guess, or "crack," but all keys had the disadvantage that they must somehow be communicated between sender and recipient. On the way, they could be stolen.

In the 1970s, a different system of cryptography was invented. In this system, there are two different keys, one to encode, or encrypt, and the other to decode, or decrypt. The encryption key can be public; only the decryption key need be kept private, and only one party needs to have it. This is the kind of system currently in widest use. It was about this time that the internet began to gain importance, and it has become one of the largest users of encryption. It turns out that Number Theory, specifically the branch involving prime numbers, is the major tool in making these encryptions systems work. Number Theory is not so useless after all.

DISCUSSION

Our Heritage(UGC Care Listed)
ISSN: 0474-9030 Vol-68, Special Issue-12

National Conference on Recent Trends in Physics, Chemistry and Mathematics (RTPCM-2020)
Held on 4th February 2020Organised by: Department of Physics, Chemistr and Mathematics, SunderraoSolanke Mahavidhyalaya, Majalgaon, MS

Interesting facts

1. All 4 digit palindromic numbers are divisible by 11.
2. If we repeat a three-digit number twice, to form a six-digit number. The result will will be divisible by 7, 11 and 13, and dividing by all three will give your original three-digit number.
3. A number of form 2^N has exactly N+1 divisors. For example 4 has 3 divisors, 1, 2 and 4.
4. To calculate sum of factors of a number, we can find the number of prime factors and their exponents. Let $p_1, p_2, \dots p_k$ be prime factors of n. Let $a_1, a_2, ..a_k$ be highest powers of $p_1, p_2, .. p_k$ respectively that divide n, i.e., we can write n as $\mathbf{n = (p_1^{a_1})*(p_2^{a_2})* \dots (p_k^{a_k})}$.

$$\text{Sum of divisors} = (1 + p_1 + p_1^2 \dots p_1^{a_1}) * (1 + p_2 + p_2^2 \dots p_2^{a_2}) * \dots\dots (1 + p_k + p_k^2 \dots p_k^{a_k})$$

We can notice that individual terms of above formula are Geometric Progressions (GP). We can rewrite the formula as.

$$\text{Sum of divisors} = (p_1^{a_1+1} - 1)/(p_1 -1) * (p_2^{a_2+1} - 1)/(p_2 -1) * \dots\dots (p_k^{a_k+1} - 1)/(p_k -1)$$

5. For a product of N numbers, if we have to subtract a constant K such that the product gets its maximum value, then subtract it from a largest value such that largest value-k is greater than 0. If we have to subtract a constant K such that the product gets its minimum value, then subtract it from the smallest value where smallest value-k should be greater than 0
6. Goldbech`s conjecture: Every even integer greater than 2 can be expressed as the sum of 2 primes.
7. Perfect numbers or Amicable numbers: Perfect numbers are those numbers Lychrel numbers: Are those numbers that cannot form a palindrome when repeatedly reversed and added to itself. For example 47 is not a Lychrel Number as 47 + 74 = 121
8. Lemoine's Conjecture : Any odd integer greater than 5 can be expressed as a sum of an odd prime (all primes other than 2 are odd) and an even semiprime. A semiprime number is a product of two prime numbers. This is called Lemoine's conjecture.
9. Fermat's Last Theorem : According to the theorem, no three positive integers a, b, c satisfy the equation, for any integer value of n greater than 2. For n = 1 and n = 2, the equation have infinitely many solutions.
10. which are equal to the sum of their proper divisors. Example: 6 = 1 + 2 + 3

GEMS BEHIND NUMBER THEORY

Pythagoras

According to tradition, Pythagoras (c. 580–500 BC) worked in southern Italy amid devoted followers. His philosophy enshrined number as the unifying concept necessary for understanding everything from planetary motion to musical harmony. Given this viewpoint, it is not surprising that the Pythagoreans attributed quasi-rational properties to certain numbers.

For instance, they attached significance to perfect numbers—i.e., those that equal the sum of their proper divisors. Examples are 6 (whose proper divisors 1, 2, and 3 sum to 6) and 28 (1 + 2 + 4 + 7 + 14). The Greek philosopher Nicomachus of Gerasa (flourished c. AD 100), writing centuries after Pythagoras but clearly in his philosophical debt, stated that perfect numbers represented “virtues, wealth, moderation, propriety, and beauty.” (Some modern writers label such nonsense numerical theology.)

Euclid

By contrast, Euclid presented number theory without the flourishes. He began Book VII of his Elements by defining a number as “a multitude composed of units.” The plural here excluded 1; for Euclid, 2 was the smallest “number.” He later defined a prime as a number “measured by a unit alone” (i.e., whose only proper divisor is 1), a composite as a numberthat is not prime, and a perfect number as one that equals the sum of its “parts” (i.e., its proper divisors).

From there, Euclid proved a sequence of theorems that marks the beginning of number theory as a mathematical (as opposed to a numerological) enterprise. Four Euclidean propositions deserve special mention.

The first, Proposition 2 of Book VII, is a procedure for finding the greatest common divisor of two whole numbers. This fundamental result is now called the Euclidean algorithm in his honor.

Second, Euclid gave a version of what is known as the unique factorization theorem or the fundamental theorem of arithmetic. This says that any whole number can be factored into the product of primes in one and only one way. For example, $1,960 = 2 \times 2 \times 2 \times 5 \times 7 \times 7$ is a decomposition into prime factors, and no other such decomposition exists. Euclid’s discussion of unique factorization is not satisfactory by modern standards, but its essence can be found in Proposition 32 of Book VII and Proposition 14 of Book IX.

Third, Euclid showed that no finite collection of primes contains them all. His argument, Proposition 20 of Book IX, remains one of the most elegant proofs in all of mathematics. Beginning with any finite collection of primes—say, a, b, c, …, n—Euclid considered the number formed by adding one to their product: N = (abc⋯n) + 1

Fourth, Euclid ended Book IX with a blockbuster: if the series 1 + 2 + 4 + 8 + … + 2k sums to a prime, then the number N = 2k(1 + 2 + 4 + … + 2k) must be perfect. For example, 1 + 2 + 4 = 7, a prime, so 4(1 + 2 + 4) = 28 is perfect. Euclid's "recipe" for perfect numbers was a most impressive achievement for its day.

Diophantus

Of later Greek mathematicians, especially noteworthy is Diophantus of Alexandria (flourished c. 250), author of Arithmetic. This book features a host of problems, the most significant of which have come to be called Diophantine equations. These are equations whose solutions must be whole numbers. For example, Diophantus asked for two numbers, one a square and the other a cube, such that the sum of their squares is itself a square. In modern symbols, he sought integers x, y, and z such that (x2)2 + (y3)2 = z2. It is easy to find real numbers satisfying this relationship (e.g., x = Square root of√2, y = 1, and z = Square root of√5), but the requirement that solutions be integers makes the problem more difficult. (One answer is x = 6, y = 3, and z = 45.) Diophantus's work strongly influenced later mathematics.

REFERENCE

- Apostol, Tom M. (1976). Introduction to analytic number theory. Undergraduate Texts in Mathematics. Springer. ISBN 978-0-387-90163-3. Retrieved 2016-02-28.
- An introduction to the theory of numbers by G. H. Hardy and E. M. Wright. Published by Oxford at the Clarendon Press.
- A classical introduction to modern number theory, second edition, by Kenneth Ireland and Michael Rosen.
- Elementary Number Theory and its Applications by Kenneth H. Rosen, 3rd ed., Addison-Wesley. Rosen