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On Neutrosophic Triplet Metric Space

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Abstract

This paper represents neutrosophic triplet v-generalized metric space and neutrosophic triplet partial metric space.

Keywords: Neutrosophic Triplet Metric Space.

Introduction

In 2018, Sahin and Kargin [58] defined neutrosophic triplet theory in neutrosophy, v-generalized metric and neutrosophic triplet v-generalized metric space.

In 2018, Sahin, Kargin and Coban [59] derived concept of neutrosophic triplet partial metric space, neutrosophic partial metric space.

Theorem (1.1):-The definition of NTVGM, if u'_1= u'_2= … = u'_v, then every neutrosophic triplet v-generalized metric implies a neutrosophic triplet metric.

Proof.$((Y,\Diamond), d_v)$ be an NTVGMS. It is clear that

1) a ◊ b ∈Y;
2) $dv(a,b) \geq 0$;
3) If $a = b, d_v(a,b) = 0$;
4) $d_v(a,b) = d_v(b,a)$

above four conditions are equal in NTVGMS and NTMS. Then for condition (5), we takeu'_1= u'_2= … = u'_v. By definition of NTVGM, If there exists elements u'_1, …, u'_v∈Y such that

$d_v(x,y) \leq d_v(x, y \Diamond neut(u'_v))$,

$d_v(x,u'_2) \leq d_v(x, u'2 \Diamond neut(u'_1))$,

$d_v(u'_1, u'_3) \leq d_v(u'_1, u'_3 \Diamond neut(u'_2))$,

⋮

ISSN: 0474-9030
Vol-68-Issue-01-January-2020

$d_v(u'_{v-1}, y) \leq d_v(u'_{v-1}, y \lozenge neut(u'_v))$;

Then,

$d_v(x, y \lozenge neut(u'_v)) \leq d_v(x, u'_1) + d_v(u'_1, u'_2) + \cdots + d_v(u'_{v-1}, u'_v) + d_v(u'_v, y)$.

Wecan take $u'_1 = u'_3 = \ldots = u'_v = u'_2$since for $u'_1 = u'_2 = \ldots = u'_v$. Thus, we can take $d_v(x, y \lozenge neut(u'_v)) = d_v(x, y \lozenge neut(u'_2))$ and

$d_v(x, u'_1) + d_v(u'_1, u'_2) + \cdots + d_v(u'_{v-1}, u'_v) + d_v(u'_v, y) = d_v(x, u'_2) + d_v(u'_2, u'_2) + \ldots + d_v(u'_2, u'_2) + d_v(u'_2, y)$.

Hence,

$$\begin{aligned} d_v(x, y \lozenge neut(u'_2)) &\leq d_v(x, u'_1) + d_v(u'_1, u'_2) + \ldots + d_v(u'_{v-1}, u'_v) + d_v(u'v, y) \\ &= d_v(x, u'_2) + d_v(u'_2, u'_2) \ldots + d_v(u'_2, u'_2) + d_v(u'_2, y) \\ &= d_v(x, u'_2) + 0 + \ldots + 0 + d_v(u'_2, y) \\ &= d_v(x, u'_2) + d_v(u'_2, y). \end{aligned}$$

As a result,$(X, \lozenge), d_v)$ is a neutrosophic triplet metric space.

Theorem (1.2):-Let $((Y, \lozenge), d_v)$be an NTVGM and there exists $u'_1, \ldots, u'_v \in$Y s. t.

$$\begin{aligned} &d_v(x, y) \leq d_v(x, y \lozenge neut(u'_v)), \\ &d_v(x, u'_2) \leq d_v(x, u'_2 \lozenge neut(u'_1)), \\ &d_v(u'_1, u'_3) \leq d_v(u'_1, u'_3 \lozenge neut(u'_2)), \\ &\qquad\vdots \\ &d_v(u'_{v-1}, y) \leq d_v(u'_{v-1}, y \lozenge neut(u'_v)), \end{aligned}$$

Where x, $u'_1, \ldots, u'_v$, y are all different. If $\{a_n\}$ is k-Cauchy sequence and converges to z $\in \{a_n\}$, then $\{a_n\}$ is a Cauchy sequence, where $a_n$are all different.

Proof. We assume that $\{a_n\}$ is k-Cauchy sequence and converges to $z_1 \in \{a_n\}$. So, $d_v(\{a_n\}, \{a_{n+1+lm}\}) < \varepsilon$ $(l = 0, 1, 2, \ldots)$. If we take $l = 0$, then

$$d_v(\{a_n\}, \{a_{n+1}\}) < \varepsilon \tag{1}$$

and

$$d_v(z, \{a_n\}) < \varepsilon \tag{2}$$

Also, we assume that there exist $u'_1, \ldots, u'_v \in N$ such that

$$\begin{aligned} &d_v(x, y) \leq d_v(x, y \lozenge neut(u'_v)), \\ &d_v(x, u'_2) \leq d_v(x, u'_2 \lozenge neut(u'_1)), \\ &d_v(u'_1, u'_3) \leq d_v(u'_1, u'_3 \lozenge neut(u'_2)), \\ &\qquad\vdots \\ &d_v(u'_{v-1}, y) \leq d_v(u'_{v-1}, y \lozenge neut(u'_v)), \end{aligned}$$

where x, $u'_1, \ldots, u'_v$, y all are different. Thus, from (v) in Definition of NTVGM, (1) and (2)

ISSN: 0474-9030
Vol-68-Issue-01-January-2020

$$\mathrm{d_v}(\{a_n\},\{a_m\}) \le \mathrm{d_v}(\{a_n\},z_1) + \mathrm{d_v}(z1,\{a_{n+v-1}\}) + \mathrm{d_v}(\{a_{n+v-1}\},\{a_{n+v-2}\}) \\ \vdots$$

$$\mathrm{d_v}(\{a_{m+1}\},\{a_m\}) \le (v-1)\varepsilon.$$

Hence, $\{a_n\}$ is a Cauchy sequence.

Theorem (1.3):-Let $((N,\lozenge),d_v)$be a neutrosophic triplet v-generalized metric space and there exist $u'_1, \ldots, u'_v$∈N such that

$$d_v(x,y) \le d_v(x,y \lozenge neut(u'_v))\text{and}$$
$$d_v(x,u'_2) \le d_v(x,u'_2 \lozenge neut(u'_1)),$$
$$d_v(u'_1,u'_3) \le d_v(u'_1,u'_3 \lozenge neut(u'_2)),$$
$$\vdots$$
$$d_v(u'_{v-1},y) \le d_v(u'_{v-1},y \lozenge neut(u'_v)).$$

If $\{a_n\}$ is sequence and $\sum_{k=1}^{\infty} d_v(a_k,a_{k+1})<\infty, k = 1 < \infty$then $\{a_n\}$ is k-Cauchy sequence, where $a_n$are all different and x, $u'_1,\ldots, u'_v$, y are all different.

Proof. Let $\varepsilon > 0$ such that $\sum_{K=1}^{\infty} d_v(a_m,a_{m+1}) <\boldsymbol{\varepsilon}$since for $\sum_{K=1}^{\infty} d_v(a_k,a_{k+1})< \infty$.Now we show that

$$d_v(\{a_n\},\{a_{n+1+lm}\}) < \varepsilon.$$

If $l = 0$, then it is clear that $(\{a_n\}, \{a_{n+1}\}) <\varepsilon$since for

$$\sum_{r=1}^{\infty} d_v(a_r,a_{r+1}) <\varepsilon.$$

We assume that $l\neq 0$. Also,

$$d_v(\{a_n\},\{a_{n+1+lm}\}) \le d_v(\{a_n\},\{a_{n+1}\}) + d_v(\{a_{n+1}\},\{a_{n+2}\}) + d_v(\{a_{n+2}\}, \{a_{n+3}\}) + \cdots + d_v(\{a_{(n+1+lm-1)}\}, a_{(n+1+lm)}\})$$
$$= \sum_{r=n}^{n+1+lm-1} d_v(a_r,a_{r+1})< \sum_{r=1}^{\infty} d_v(a_r,a_{r+1})< \varepsilon.$$

Hence, a_n is k-Cauchy sequence.

Theorem (1.4):-Take a non-empty S set and power set of S is P(S) and m(A) be cardinal of $A \in P(S)$ and $(P(S),\lozenge), d)$ be a NT metric space (NTMS). If there exists any $K \in P(S)$ such that $m(B \lozenge neut(K)) = m(B)$. Then, $((P(S),\lozenge),p_N)$ is a NTPMS such that

$$p_N(\mathrm{A, B}) = \frac{\mathrm{d(A,B)+m(A)+m(B)}}{2}.$$

Proof.

(i) $p_N(A,A) = \frac{\mathrm{d(A,A)+m(A)+m(A)}}{2} = m(A) \le \frac{\mathrm{d(A,B)+m(A)+m(B)}}{2}$

ISSN: 0474-9030
Vol-68-Issue-01-January-2020

$= p_N(A,B)$, since for $d(A,A) = 0$.

$0 \le p_N(A,A) \le p_N(A,B)$ $for\ A,B \in P(S)$.

(ii) If $p_N(A, A) = p_N(A, B) = (B, B) = 0$, then

(iii) $\frac{d(A,A)+m(A)+m(A)}{2} = \frac{d(A,B)+m(A)+m(B)}{2} = \frac{d(B,B)+m(B)+m(B)}{2} = 0$ and

$d(A,B) + m(A) + m(B) = 0$.Where, $m(A) = 0, m(B) = 0$ and $p_N(A,B) = 0$.

Thus, $A = B = \emptyset$.

(iv) $p_N(A, B) = \frac{d(A,B)+m(A)+m(B)}{2} = \frac{d(B,A)+m(B)+m(A)}{2} = p_N(B, A)$, since for

$p_N(A,B) = p_N(B,A)$.

(v) We suppose that there exists a $K \in P(S)$ such that $(B \lozenge neut(K)) = m(B)$ and $p_N(A,B) \le p_N(A, B \lozenge neut(K))$.

Thus,

$$\frac{d(A,B) + m(A) + m(B)}{2} \le \frac{d(A,B\lozenge neut(K)) + m(A) + m(B\lozenge neut(K))}{2} \quad (1)$$

From (1), $p_N(A,B) \le p_N(A,B \lozenge neut(K))$. Since $(P(S), \lozenge)$, d) is a NTMS,

$$d(A,B \lozenge neut(K)) \le d(A,K) + d(K,B) \quad (2)$$

From (1), (2) we have,

$$\frac{d(A,B)+m(A)+m(B)}{2} \le \frac{d(A,B\lozenge neut(K)) + m(A) + m(B\lozenge neut(K))}{2}$$

$$\le \frac{d(A,K)+ d(K,B)+m(A)+m(B)}{2}$$

$$= \frac{d(A,K) + m(A) + m(K)}{2} + \frac{d(K,B) + m(B) + m(K)}{2} - m(K).$$

Where, $p_N(K, K) = m(K)$.

Thus,

$$p_N(A,B \lozenge neut(K)) \le (A,K) + (K,B) - (K,K).$$

Hence,

$((P(S),\lozenge), pN)$ is a NTPMS.

Theorem (1.5):-Let $(X,\lozenge)$ be a NT set, $k \in R^+$ and $((X,\lozenge), d_T)$ be a NTMS. Then; $((X,\lozenge), p_N)$ is a NTPMS such that $(x,y) = (x,y) + k, \forall x,y \in X$.

Proof.

(i) Since for $(x, x) = 0$,

$0 \le p_N(x,x) = d_T(x,x) + k = k \le p_N(x,y) = d_T(x,y) + k$. Thus;

(ii) $0 \le p_N(x,x) \le p_N(x,y)$.

ISSN: 0474-9030
Vol-68-Issue-01-January-2020

(iii) There does not exists $x, y \in A$ such that

$p_N(x,x) = p_N(x,y) = p_N(y,y) = 0$ since for $k \in \mathbb{R}^+$ and $d_T(x,x) = 0$.

(iv) $p_N(x,y) = d_T(x,y) + k = d_T(y,x) + k$, since for $d_T(x,y) = d_T(y,x)$.

(v) Suppose that there exists any element $q \in A$ such that

$p_N(x,y) \leq p_N(x, y \lozenge neut(q))$. Then $p_N(x,y) + k \leq p_N(x, y \lozenge neut(q)) + k$.

Thus, $p_N(x,y) \leq p_N(x, y \lozenge neut(q))$ (3)

Also,

$$p_N(x, y \lozenge neut(q)) \leq p_N(x,q) + p_N(q,y) \quad (4)$$

Since for $((A, \lozenge), d_T)$ is a NTMS.

From (3) and (4),

$p_N(x,y) \leq p_N(x, y \lozenge neut(q)) = d_T(x, y \lozenge neut(q)) + k$

$\leq d_T(x,q) + p_N(q,y) + k$

$= p_N(x,q) + p_N(q,y) - k$. Where, $p_N(q,q) = k$.

Thus,

$$p_N(x, y \lozenge neut(q)) \leq p_N(x,q) + p_N(q,y) - (q,q).$$

Hence, $((A,\lozenge), p_N)$ is a NTPMS.

Theorem (1.6):- Let $((X,\lozenge), p_N)$ be a NTPMS, $\{a_n\}$ be a convergent sequence in NTPMS and $p_N(\{a_m\},\{a_n\}) \leq p_N(\{a_m\},\{a_n\} \lozenge neut(u))$ for any $u \in A$. Then $\{a_n\}$ is a Cauchy sequence in NTPMS.

Proof. It is clear that

$$p_N(u,\{x_n\}) < \frac{\varepsilon}{2} + p_N(u,u) \quad (5)$$

for each $M \leq n$ or

$$p_N(u,\{x_m\}) < \frac{\varepsilon}{2} + p_N(u,u) \quad (6)$$

for each $M \leq m$

Because $\{x_n\}$ is a convergent. Then, we suppose that

$p_N(\{x_m\},\{x_n\}) \leq p_N(\{x_m\},\{x_n\}) \lozenge neut(a))$ for any $u \in A$. It is clear that for $n, m \geq M$,

$$p_N(\{x_m\},\{x_n\}) \leq p_N(\{x_m\},\{x_n\} \lozenge neut(u))$$
$$\leq p_N(u,\{xn\}) + p_N(u,\{x_m\}) - p_N(u,u) \quad (7)$$

Because $((A,\lozenge), pN)$ is a NTPMS. From (5), (6) and (7),

$$p_N(\{x_m\},\{x_n\}) < \frac{\varepsilon}{2} + p_N(u,u) + \frac{\varepsilon}{2} + p_N(u,u) - p_N(u,u) = \varepsilon + p_N(u,u).$$

Thus; $\{x_n\}$ is a Cauchy sequence in $((A,\lozenge), p_N)$.

ISSN: 0474-9030
Vol-68-Issue-01-January-2020

Theorem (1.7):-For each contraction m over a complete NTPMS $((X,\Diamond),p_N)$, there exists a unique a in X such that $a = m(a)$. Also, $p_N(a,a) = 0$.

Proof. Let $m$be a contraction for $((A,\Diamond),p_N)$ complete NTPMS and $x_n = m(x_{n-1})$ and $x_0 \in A$be a unique element. Also, we can take

$$p_N(x_n,x_k) \le p_N(x_n,x_k \Diamond neut(x_{n-1})) \quad (8)$$

Since for $m$is a contraction over $((A,\Diamond),p_N)$ complete NTPMS. Then,

$p_N(x_2,x_1) = p_N(m(x_1),m(x_0)) \le c.p_N(x_1,x_0)$ and

$p_N(x_3,x_2) = p_N(m(x_2),m(x_1)) \le c.p_N(x_2,x_1) \le c^2.p_N(x_1,x_0)$.

From mathematical induction, $n \ge m$;

$$\begin{aligned} p_N(x_{m+1},x_m) &= p_N(m(x_m),m(x_{m-1})) \\ &\le c.p_N(x_m,x_{m-1}) \le c^m.p_N(x_1,x_0). \end{aligned}$$

Thus; from (8) and definition of NTPMS,

$$\begin{aligned} p_N(x_n,x_m) &\le p_N(x_n,x_m \Diamond neut(x_{n-1})) \\ &\le (x_n,x_{n-1}) + p_N(x_{n-1},x_m) - p_N(x_{n-1},x_{n-1}) \\ &\le c^{n-1}.(x_1,x_0) + (x_{n-1},x_m) - (x_{n-1},x_{n-1}) \\ &\le c^{n-1}.(x_1,x_0) + (x_{n-1},x_{n-2}) + \ldots + (x_m,x_{m-1}) \\ &\le (c^{n-1}+c^{n-2}+\ldots+c^{m-1}+c^m).(x_1,x_0) - \textstyle\sum_{i=m}^{n-1} p_N(x_i,x_i) \\ &\le \textstyle\sum_{i=m}^{n-1} c^i p_N(x_1,x_0) - \sum_{i=m}^{n-1} p_N(x_i,x_i) \\ &\le \textstyle\sum_{i=m}^{n-1} c^i p_N(x_1,x_0) - \sum_{i=m}^{n-1} p_N(x_0,x_0) \\ &= \textstyle\sum_{i=m}^{n=1} c^i p_N(x_1,x_0) - \sum_{i=m}^{n=1} p_N(x_i,x_i) \ (For\ n,m \to \infty) \\ &= \frac{c^m}{1-c} p_N(x_1,x_0) + p_N(x_0,x_0) \to p_N(x_0,x_0) \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence. Also $\{x_n\}$ is convergent such that $x_n \to x$. Because $((A,\Diamond),p_N)$ is complete NTPMS. Thus; $m(x_n) \to m(x)$ since for $x_n = m(x_{n-1})$; $m(x_n) = x_{n+1} \to x$. Thus,$m(x) = x$.Suppose that $m(x) = x$or $m(y) = y$for $x,y \in x_n$. Where,

$$p_N(x,y) = p_N(m(x),m(y)) \le c.\,p_N(x,y).\,p_N(x,y) > 0, c \ge 1.$$

It is a contradiction. Thus; $p_N(x,y) = p_N(x,x) = p_N(y,y) = 0$ and $x = y$.

Therefore, $p_N(x,x) = 0$.

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ISSN: 0474-9030
Vol-68-Issue-01-January-2020

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ISSN: 0474-9030
Vol-68-Issue-01-January-2020

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